Shell-Beam FE Model in Static and Stability Analysis of Thin-Walled Structures with Open Cross-Section

Mathematical models for the static and stability analysis of elastic spatial thin-walled structures with open cross-section were formulated in the doctoral thesis. In the structure, parts treated as 3D (shell) geometric objects (frame joints, supports, sites where loads are applied, sites with additional bracing) and the remaining ones, considered as 1D (beam) geometrically linear objects, were specified. Numerical solutions were obtained using Finite Element Method (FEM) with the Total Lagrangian Approach. While computing matrices for finite elements, Reisner–Mindlin plate theory and Vlasov thin-walled beam theory were employed. FEM equilibrium governing equations were solved with the Newton–Raphson method, controlling the load parameter or a selected displacement.

Two analysis methods were developed. In the first method, the equations accounting for the continuity of displacements between the 3D and 1D structure parts, were treated as constraints equations and included in FEM equilibrium equations due to the properly defined penalty function. As a result, it was necessary to include the so-called *transition elements* between the shell nodes and beam nodes. The method is general and can be applied to solving both linear and non-linear problems. In the study, the method was used to analyse the stability of thin-walled structures.

In the other method, 3D parts of the structure were discreted with the shell finite elements and condensed to the so-called *space joint elements*

with translational degrees of freedom on the walls, which connected those elements to the adjacent thin-walled beams. Finally, the degrees of freedom of space joint elements were transformed to thin-walled beam degrees of freedom. Such a FEM discretization model proved to be particularly effective for the linear analysis of frames with different, complex cross-sections.

The models described above were implemented into the AmFEM computer program, developed by the author, which was running under MATLAB computing software. Additionally, SecPropGRAPH and SecPropFEM programs were written that facilitated computing the geometric characteristics of the thin-walled element cross-section, which were not available in the literature on the subject. A lot of attention was given to devising procedures for the calculations of tangent matrices and internal forces vector for the thin-walled element, which in accordance with the assumptions adopted, were computed accurately. Symbolic calculations (provided by MATLAB software) and PERL programming language, supporting operations that employ regular expressions, were used.

Many examples were performed to illustrate major features of the proposed analysis methods and to indicate the effectiveness of the solutions put forward. In the author's opinion, the methods can help with linear and geometrically non-linear analyses of spatial thin-walled structures with arbitrarily complex open cross-sections.