

The SecPropFEM program

The SecPropFEM program computes the geometric properties of thin-walled open cross-sections. It was written and tested in MATLAB 7.3.0 (R2006b). The program is a function with the syntax:

<code>Section = SecPropFEM(Nodes, Elements, WarpNodes)</code>

where:

- `Nodes` – array containing the consecutive node numbers and their coordinates (y, z) ,
- `Elements` – connectivity array defining the quadrilateral elements,
- `WarpNodes` – array of selected node numbers (may be empty); for these nodes the coordinates in the principal centroidal system and the warping function values are computed,
- `Section` – output structure.

The program solves the Neumann problem

$$\nabla^2 \omega(y, z) \equiv \frac{\partial^2 \omega(y, z)}{\partial y^2} + \frac{\partial^2 \omega(y, z)}{\partial z^2} = 0, \quad (1)$$

with the natural boundary condition on the cross-section contour Γ with the outward unit normal $\mathbf{e}_n = \{e_{n_y} \ e_{n_z}\}$ of the normal $\mathbf{n}$

$$\left(\frac{\partial \omega(y, z)}{\partial y} - z \right) e_{n_y} + \left(\frac{\partial \omega(y, z)}{\partial z} + y \right) e_{n_z} = 0, \quad (2)$$

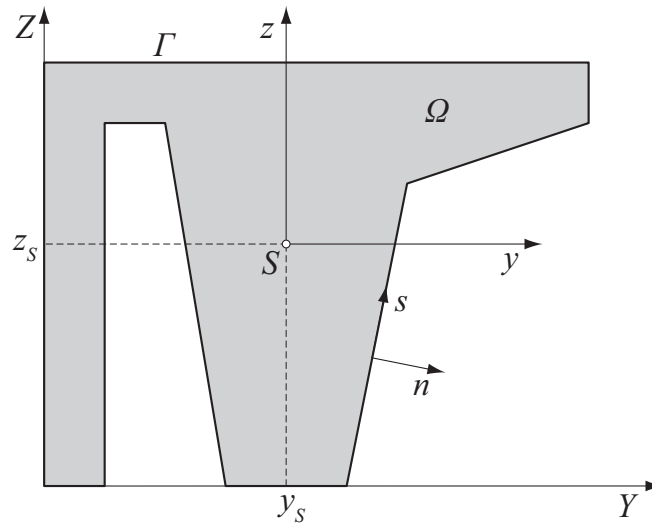
and the essential boundary condition

$$\omega(y_S, z_S) = 0 \quad (3)$$

at the shear centre S with coordinates (y_S, z_S) .

Example script:

<pre>Nodes = [1 0 0 2 10 0 3 0.5 0.4679 ... 1031 9.5517 0.8977]; Elements = [1 304 305 484 482 2 305 1 306 484 3 482 484 485 3 ... 869 1031 340 341 1027]</pre>				
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Rys. 1. Cross-section of a thin-walled profile

```
];
```

```
MarkNodes = [6 5 92 3 4];
```

```
Section = SecPropFEM(Nodes, Elements, MarkNodes)
```

As a result, the program returns the Section structure:

```
Section =
    Area: 59.0995
    Sy: 2.3358e-012
    Sz: 3.3640e-013
    So: -8.0602e-012
    Jy: 8.0853e+003
    Jz: 493.9747
    Jyz: -6.5694e-010
    Jr: 8.5793e+003
    Jt: 39.2345
    Jo: 6.8183e+004
    Jyo: 3.4987e-009
    Jzo: -1.5277e-012
    Jry: 1.0099e+004
    Jrz: 1.1253e+003
    Jro: 352.0739
    UserYc: 2.7050
    UserZc: 14.9184
    UserYs: -2.6243
    UserZs: 14.9593
    Ys: -5.3292
    Zs: 0.0462
    Gamma: 9.9697e-004
    Transform: [2x2 double]
    WarpNodes: [5x4 double]
    My4: 1.0903e+004
```

Mz4: 1.4808e+006
Mo4: 2.0518e+008
My3: 1.4323e+003
Mz3: 1.1066e+003
Mo3: 2.4420e+004
Moy2: 8.0718e+003
Moz2: 2.1468e+003
Moy: -2.7319
Moz: -314.9520
My3z: 175.1253
My3o: 654.6312
Mz3y: 1.9298e+003
Mz3o: 5.1253e+005
Mo3y: 1.9469e+005
Mo3z: 2.3138e+007
My2z2: 8.2034e+004
My2o2: 1.4098e+006
My2oy2: 8.1960e+004
My2oz2: 3.9380e+004
My2z: 18.7441
My2o: 113.1077
My2oy: -4.0762
My2oz: -4.0549e+003
Mz2o2: 1.2342e+007
Mz2oy2: 1.4663e+006
Mz2oz2: 3.9891e+005
Mz2y: 8.6669e+003
Mz2o: 238.9662
Mz2oy: 745.0466
Mz2oz: -5.1755e+004
Mo2oy2: 1.2385e+007
Mo2oz2: 4.9927e+006
Mo2y: 1.6276e+005
Mo2z: 8.7164e+003
Mo2oy: 5.5594e+003
Mo2oz: -5.2791e+005
Myoy2: 8.6934e+003
Myoz2: 6.4417e+003
Mzoy2: 374.2829
Mzoz2: 29.7665
Myoy: 3.2428e-004
Myoz: -474.3565
Mzoy: 8.0657e+003
Mzoz: 0.0035
Mooy2: 155.6904
Mooz2: 190.9111
Mooy: 109.2331
Mooz: -7.0502
My2zo: 2.4241e+005
Mz2yo: 1.1355e+004
Mo2yz: 4.0196e+004
Myzoy2: 1.1321e+003

Myzoz2:	451.5591
Myzo:	7.0166e+004
Myzoy:	8.6619e+003
Myzoz:	-24.8622
Myooy2:	4.8034e+003
Myooz2:	1.9853e+003
Mzooy2:	5.5781e+005
Mzooz2:	9.4859e+005
Myooy:	7.0318e+004
Myooz:	-112.2013
Mzooy:	201.2043
Mzooz:	-6.6912e+004

The following notation is used in the formulas below:

a – cross-sectional area,

ω – warping function,

y, z – coordinates of a point in the cross-section.

The output structure Section contains the following fields:

Area – cross-sectional area of the thin-walled profile,

$$A = \iint_a dy \, dz, \quad (4)$$

S_y – first moment of area about the y axis,

$$S_y = \iint_a z \, dy \, dz, \quad (5)$$

S_z – first moment of area about the z axis,

$$S_z = \iint_a y \, dy \, dz, \quad (6)$$

S_ω – first moment of warping,

$$S_\omega = \iint_a \omega \, dy \, dz, \quad (7)$$

J_y – second moment of area about the y axis,

$$J_y = \iint_a z^2 \, dy \, dz, \quad (8)$$

J_z – second moment of area about the z axis,

$$J_z = \iint_a y^2 \, dy \, dz, \quad (9)$$

J_{yz} – product of inertia,

$$J_{yz} = \iint_a y \cdot z \, dy \, dz, \quad (10)$$

J_r – polar moment of inertia,

$$J_r = \iint_a (y^2 + z^2) \, dy \, dz, \quad (11)$$

Jt – torsion constant,

$$J_t = \iint_a \left(y^2 + z^2 + y \cdot \frac{\partial \omega}{\partial z} - z \cdot \frac{\partial \omega}{\partial y} \right) dy dz, \quad (12)$$

Jo – warping constant,

$$J_\omega = \iint_a \omega^2 dy dz, \quad (13)$$

Jyo – warping product of inertia,

$$J_{y\omega} = \iint_a y \cdot \omega dy dz, \quad (14)$$

Jzo – warping product of inertia,

$$J_{z\omega} = \iint_a z \cdot \omega dy dz, \quad (15)$$

Jry – higher-order moment,

$$J_{ry} = \iint_a (y^2 + z^2) \cdot y dy dz, \quad (16)$$

Jrz – higher-order moment,

$$J_{rz} = \iint_a (y^2 + z^2) \cdot z dy dz, \quad (17)$$

Jro – higher-order moment,

$$J_{r\omega} = \iint_a (y^2 + z^2) \cdot \omega dy dz, \quad (18)$$

UserYc, UserZc – coordinates (y_C, z_C) of the centroid in the user coordinate system,

UserYs, UserZs – coordinates (y_S, z_S) of the shear centre in the user coordinate system,

UserYq, UserZq – coordinates (y_Q, z_Q) of the principal point of zero sectorial coordinate in the user coordinate system,

Ys, Zs – coordinates (Y_S, Z_S) of the shear centre in the principal centroidal axes,

Yq, Zq – coordinates (Y_Q, Z_Q) of the principal point of zero sectorial coordinate in the principal centroidal axes,

Gamma – angle γ of the transformation from the user coordinate system to the principal axes,

Transform – transformation matrix from the user coordinate system to the principal axes,

WarpNodes – coordinates of the selected nodes and the warping function values at these nodes, in the principal centroidal axes,

My4 – higher-order moment,

$$M_{y^4} = \iint_a y^4 dy dz, \quad (19)$$

Mz4 – higher-order moment,

$$M_{z^4} = \iint_a z^4 dy dz, \quad (20)$$

Mo4 – higher-order moment,

$$M_{\omega^4} = \iint_a \omega^4 \, dy \, dz, \quad (21)$$

My3 – higher-order moment,

$$M_{y^3} = \iint_a y^3 \, dy \, dz, \quad (22)$$

Mz3 – higher-order moment,

$$M_{z^3} = \iint_a z^3 \, dy \, dz, \quad (23)$$

Mo3 – higher-order moment,

$$M_{\omega^3} = \iint_a \omega^3 \, dy \, dz, \quad (24)$$

Moy2 – higher-order moment,

$$M_{\omega, y^2} = \iint_a \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (25)$$

Moz2 – higher-order moment,

$$M_{\omega, z^2} = \iint_a \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (26)$$

Moy – higher-order moment,

$$M_{\omega, y} = \iint_a \frac{\partial \omega}{\partial y} \, dy \, dz, \quad (27)$$

Moz – higher-order moment,

$$M_{\omega, z} = \iint_a \frac{\partial \omega}{\partial z} \, dy \, dz, \quad (28)$$

My3z – higher-order moment,

$$M_{y^3 z} = \iint_a y^3 \cdot z \, dy \, dz, \quad (29)$$

My3o – higher-order moment,

$$M_{y^3 \omega} = \iint_a y^3 \cdot \omega \, dy \, dz, \quad (30)$$

Mz3y – higher-order moment,

$$M_{z^3 y} = \iint_a z^3 \cdot y \, dy \, dz, \quad (31)$$

Mz3o – higher-order moment,

$$M_{z^3 \omega} = \iint_a z^3 \cdot \omega \, dy \, dz, \quad (32)$$

Mo3y – higher-order moment,

$$M_{\omega^3 y} = \iint_a \omega^3 \cdot y \, dy \, dz, \quad (33)$$

Mo3z – higher-order moment,

$$M_{\omega^3 z} = \iint_a \omega^3 \cdot z \, dy \, dz, \quad (34)$$

My2z2 – higher-order moment,

$$M_{y^2 z^2} = \iint_a y^2 \cdot z^2 \, dy \, dz, \quad (35)$$

My2o2 – higher-order moment,

$$M_{y^2 \omega^2} = \iint_a y^2 \cdot \omega^2 \, dy \, dz, \quad (36)$$

My2oy2 – higher-order moment,

$$M_{y^2 \omega, y^2} = \iint_a y^2 \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (37)$$

My2oz2 – higher-order moment,

$$M_{y^2 \omega, z^2} = \iint_a y^2 \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (38)$$

My2z – higher-order moment,

$$M_{y^2 z} = \iint_a y^2 \cdot z \, dy \, dz, \quad (39)$$

My2o – higher-order moment,

$$M_{y^2 \omega} = \iint_a y^2 \cdot \omega \, dy \, dz, \quad (40)$$

My2oy – higher-order moment,

$$M_{y^2 \omega, y} = \iint_a y^2 \cdot \frac{\partial \omega}{\partial y} \, dy \, dz, \quad (41)$$

My2oz – higher-order moment,

$$M_{y^2 \omega, z} = \iint_a y^2 \cdot \frac{\partial \omega}{\partial z} \, dy \, dz, \quad (42)$$

Mz2o2 – higher-order moment,

$$M_{z^2 \omega^2} = \iint_a z^2 \cdot \omega^2 \, dy \, dz, \quad (43)$$

Mz2oy2 – higher-order moment,

$$M_{z^2 \omega, y^2} = \iint_a z^2 \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (44)$$

Mz2oz2 – higher-order moment,

$$M_{z^2 \omega, z^2} = \iint_a z^2 \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (45)$$

Mz2y – higher-order moment,

$$M_{z^2y} = \iiint_a z^2 \cdot y \, dy \, dz, \quad (46)$$

Mz2o – higher-order moment,

$$M_{z^2\omega} = \iiint_a z^2 \cdot \omega \, dy \, dz, \quad (47)$$

Mz2oy – higher-order moment,

$$M_{z^2\omega,y} = \iiint_a z^2 \cdot \frac{\partial \omega}{\partial y} \, dy \, dz, \quad (48)$$

Mz2oz – higher-order moment,

$$M_{z^2\omega,z} = \iiint_a z^2 \cdot \frac{\partial \omega}{\partial z} \, dy \, dz, \quad (49)$$

Mo2oy2 – higher-order moment,

$$M_{\omega^2\omega,y^2} = \iiint_a \omega^2 \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (50)$$

Mo2oz2 – higher-order moment,

$$M_{\omega^2\omega,z^2} = \iiint_a \omega^2 \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (51)$$

Mo2y – higher-order moment,

$$M_{\omega^2y} = \iiint_a \omega^2 \cdot y \, dy \, dz, \quad (52)$$

Mo2z – higher-order moment,

$$M_{\omega^2z} = \iiint_a \omega^2 \cdot z \, dy \, dz, \quad (53)$$

Mo2oy – higher-order moment,

$$M_{\omega^2\omega,y} = \iiint_a \omega^2 \cdot \frac{\partial \omega}{\partial y} \, dy \, dz, \quad (54)$$

Mo2oz – higher-order moment,

$$M_{\omega^2\omega,z} = \iiint_a \omega^2 \cdot \frac{\partial \omega}{\partial z} \, dy \, dz, \quad (55)$$

Myoy2 – higher-order moment,

$$M_{y\omega,y^2} = \iiint_a y \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (56)$$

Myoz2 – higher-order moment,

$$M_{y\omega,z^2} = \iiint_a y \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (57)$$

Mzoy2 – higher-order moment,

$$M_{z\omega,y^2} = \iint_a z \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 dy dz, \quad (58)$$

Mzoz2 – higher-order moment,

$$M_{z\omega,z^2} = \iint_a z \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 dy dz, \quad (59)$$

Myoy – higher-order moment,

$$M_{y\omega,y} = \iint_a y \cdot \frac{\partial \omega}{\partial y} dy dz, \quad (60)$$

Myoz – higher-order moment,

$$M_{y\omega,z} = \iint_a y \cdot \frac{\partial \omega}{\partial z} dy dz, \quad (61)$$

Mzoy – higher-order moment,

$$M_{z\omega,y} = \iint_a z \cdot \frac{\partial \omega}{\partial y} dy dz, \quad (62)$$

Mzoz – higher-order moment,

$$M_{z\omega,z} = \iint_a z \cdot \frac{\partial \omega}{\partial z} dy dz, \quad (63)$$

Mooy2 – higher-order moment,

$$M_{\omega\omega,y^2} = \iint_a \omega \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 dy dz, \quad (64)$$

Moоз2 – higher-order moment,

$$M_{\omega\omega,z^2} = \iint_a \omega \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 dy dz, \quad (65)$$

Mooy – higher-order moment,

$$M_{\omega\omega,y} = \iint_a \omega \frac{\partial \omega}{\partial y} dy dz, \quad (66)$$

Moоз – higher-order moment,

$$M_{\omega\omega,z} = \iint_a \omega \frac{\partial \omega}{\partial z} dy dz, \quad (67)$$

My2zo – higher-order moment,

$$M_{y^2z\omega} = \iint_a y^2 \cdot z \cdot \omega dy dz, \quad (68)$$

Mz2yo – higher-order moment,

$$M_{z^2 y \omega} = \iint_a z^2 \cdot y \cdot \omega \, dy \, dz, \quad (69)$$

Mo2yz – higher-order moment,

$$M_{\omega^2 y z} = \iint_a \omega^2 \cdot y \cdot z \, dy \, dz, \quad (70)$$

Myzo2 – higher-order moment,

$$M_{y z \omega, y^2} = \iint_a y \cdot z \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (71)$$

Myzoz2 – higher-order moment,

$$M_{y z \omega, z^2} = \iint_a y \cdot z \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (72)$$

Myzo – higher-order moment,

$$M_{y z \omega} = \iint_a y \cdot z \cdot \omega \, dy \, dz, \quad (73)$$

Myzoy – higher-order moment,

$$M_{y z \omega, y} = \iint_a y \cdot z \cdot \frac{\partial \omega}{\partial y} \, dy \, dz, \quad (74)$$

Myzoz – higher-order moment,

$$M_{y z \omega, z} = \iint_a y \cdot z \cdot \frac{\partial \omega}{\partial z} \, dy \, dz, \quad (75)$$

Myooy2 – higher-order moment,

$$M_{y \omega \omega, y^2} = \iint_a y \cdot \omega \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (76)$$

Myooz2 – higher-order moment,

$$M_{y \omega \omega, z^2} = \iint_a y \cdot \omega \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (77)$$

Mzooy2 – higher-order moment,

$$M_{z \omega \omega, y^2} = \iint_a z \cdot \omega \cdot \left(\frac{\partial \omega}{\partial y} \right)^2 \, dy \, dz, \quad (78)$$

Mzooz2 – higher-order moment,

$$M_{z \omega \omega, z^2} = \iint_a z \cdot \omega \cdot \left(\frac{\partial \omega}{\partial z} \right)^2 \, dy \, dz, \quad (79)$$

Myooy – higher-order moment,

$$M_{y\omega\omega,y} = \iint_a y \cdot \omega \cdot \frac{\partial \omega}{\partial y} dy dz, \quad (80)$$

Myooz – higher-order moment,

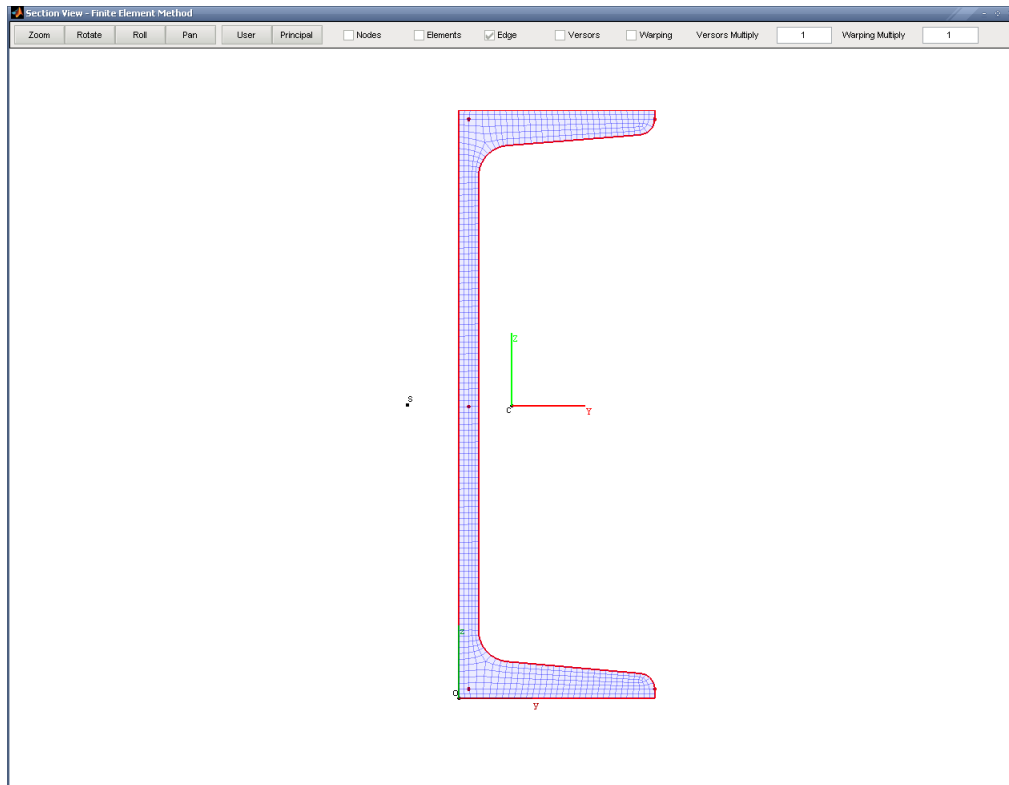
$$M_{y\omega\omega,z} = \iint_a y \cdot \omega \cdot \frac{\partial \omega}{\partial z} dy dz, \quad (81)$$

Mzoooy – higher-order moment,

$$M_{z\omega\omega,y} = \iint_a z \cdot \omega \cdot \frac{\partial \omega}{\partial y} dy dz, \quad (82)$$

Mzooz – higher-order moment,

$$M_{z\omega\omega,z} = \iint_a z \cdot \omega \cdot \frac{\partial \omega}{\partial z} dy dz. \quad (83)$$



Rys. 2. The SecPropFEM program window

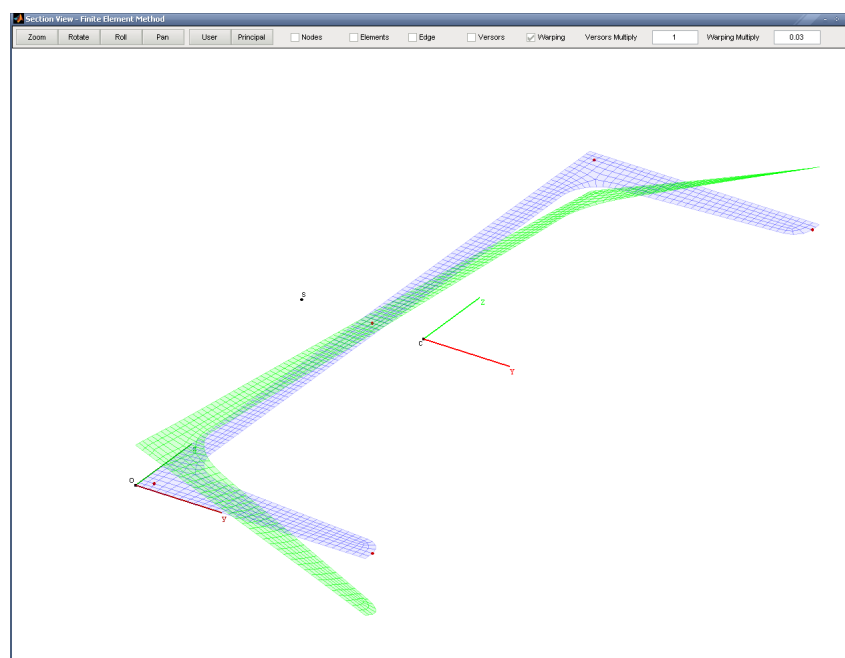
The top toolbar contains the tools:

- | | | |
|---------------|---|---|
| Zoom | – | zooming the section sketch in and out, |
| Rotate | – | free rotation of the sketch, |
| Roll | – | rotation of the sketch in the screen plane, |
| Pan | – | panning the sketch, |

<input checked="" type="button" value="User"/>	–	view in the user coordinate system,
<input checked="" type="button" value="Principal"/>	–	view in the principal centroidal coordinate system,
☐ Nodes	–	display of node numbers,
☐ Elements	–	display of element numbers,
☐ Edge	–	display of the outer boundary,
☐ Vectors	–	display of the outer boundary normals,
☐ Warping	–	display of the warping function,

The following parameters can also be changed:

Vectors Multiply	<input type="text" value="1"/>	–	scale factor used when drawing the outer boundary normals,
Warping Multiply	<input type="text" value="1"/>	–	scale factor used when drawing the warping function.



Rys. 3. The SecPropFEM program window with the warping function plotted